Inelastic scattering rates in d-wave superconductors.

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The inelastic scattering rates of quasiparticles in a two-dimensional d-wave superconductor, which arise from interactions with either acoustic phonons or other quasiparticles, are calculated within second order perturbation theory. We discover a strong enhancement of scattering with collinear momenta, brought about by the special kinematics of the two-dimensional fermions with Dirac-like spectrum near the nodes of the d-wave order parameter. In the case of a local instantaneous interparticle potential we find that either an RPA-type resummation of the perturbation series or an inclusion of non-linear corrections to the Dirac spectrum is called for in order to obtain a finite scattering rate in the limit $\omega/T \to 0$. In either way, we find drastic changes in the scattering rate, as compared to the naively expected cubic temperature dependence.

Recent measurements of microwave [1] and thermal conductivity [2] in 90 K YBCO have probed the quasiparticle relaxation rates in a range of temperatures below T_c for which inelastic scattering is believed to be important, and rates approximately proportional to T^4 and T^2, respectively, were revealed. Interestingly enough, both behaviours deviate from the simple cubic rate which would follow from the naive Golden rule estimate for the el-el scattering rate in the presence of a linear density of quasiparticle states. The likelihood of this discrepancy being due to an intricate difference between the quasiparticle lifetime versus, generally different, charge and energy relaxation rates motivates one to revisit the calculations based upon the golden rule.

We first consider the scattering of quasiparticles off three-dimensional acoustic phonons at temperatures below the Debye temperature. After having integrated the second order perturbation theory result for the inelastic quasiparticle lifetime over the out-of-plane phonon momenta, one recovers the expected cubic rate

$$\tau_{el-ph}^{-1} \sim I(1)\frac{T^3}{\theta_{Debye}\Delta_0}, \qquad (1)$$

which is frequency independent in the limit $\omega \ll T$. However, the prefactor $I(1)$ obtained from the angular part of the planar momentum inte-

gral becomes divergent when the speed of sound s approaches the value of the quasiparticle velocity v_2 parallel to the Fermi surface (the latter is proportional to the maximum superconducting gap and therefore much smaller than v_F): $I(1) \sim \ln(v_2/|v_2 - s|)$.

Although posing no real problem for the el-ph scattering rate, the above divergence reflects a potential danger in the case of el-el scattering of quasiparticles with equal velocities.

To this end, we consider a local instantaneous interaction between either charge or spin densities. In the leading approximation, one can neglect scattering processes between any pair of the neighbouring nodes because of the strong eccentricity of the equal energy contours in momentum space ($v_F/v_2 \sim 14 \pm 3$) [3]. For the remaining *intra-node* and *opposite-node* processes scaling of the local momenta near the nodes by the corresponding velocities gives rise to the symmetrical Dirac-like quasiparticle spectrum $E(\mathbf{k}) = |\mathbf{k}|$.

Further analysis reveals, that amongst the terms corresponding to different combinations of coherence factors [4] and nodes, involved in the scattering processes, the most important ones are those which correspond to scattering off thermally excited quasiparticles with nearly parallel momenta.

It proves convenient to write the scattering rate in terms of the charge or spin susceptibility

$$\begin{aligned}\chi''(\mathbf{q},\Omega) &= \frac{1}{v_1 v_2}\int\frac{d^2 k'}{(2\pi)^2}\left[f(k'-\Omega)-f(k')\right]\\ &\quad\times\delta(\Omega-k'-|\mathbf{k}'+\mathbf{q}|)\\ &\approx \frac{\theta(\Omega-q)}{64\,v_1 v_2}\frac{2\Omega^2-q^2}{\sqrt{\Omega^2-q^2}}\frac{\Omega}{T},\quad(2)\end{aligned}$$

valid for $\Omega \ll T$. Eq. (2) is then integrated with the energy conserving delta function and the appropriate combination of distribution functions, and for the external momentum taken right on the node this yields the integral

$$\begin{aligned}\tau^{-1}(\omega) &\sim \int_0^\infty dq\,\frac{q^2(2(\omega+q)^2-q^2)}{v_1 v_2 T\sqrt{\omega(\omega+2q)}}\,\mathrm{csch}\left(\frac{q}{T}\right)\\ &\sim \frac{T^{7/2}}{t\Delta_0\omega^{1/2}},\quad\text{for}\quad\omega\ll T,\quad(3)\end{aligned}$$

which tends to diverge in the limit $\omega/T \to 0$, unlike the omitted regular terms proportional to ω^3 or ωT^2. A mere substitution of ω by T would yield a cubic temperature dependence but conceal the divergence stemming from the momenta $q \approx \Omega$. This kinematical singularity, which reflects enhancement of scattering between particles with collinear momenta, has also been noted in the early studies of semi-metals [5].

In lattice models, the above divergence is readily cut off by non-linear corrections to the bare quasiparticle spectrum. In the case of parameters relevant for the problem of high temperature superconductors ($t' \sim -0.5t$) we obtain

$$\tau^{-1} \sim \left(\frac{T}{t}\right)^{3/2} T.\quad(4)$$

Nonetheless, in the case of a strictly linear spectrum no physically meaningful result can be obtained without a summation of the entire perturbation series. As an attempt to carry out this procedure one can substitute the bare interaction λ by an effective interaction

$$D(i\nu,\mathbf{q}) = \frac{\lambda}{1-\lambda\chi(i\nu,\mathbf{q})},\quad(5)$$

containing the singular susceptibility (2). In the limit $\omega/T \ll (\lambda T/t\Delta_0)^2$, this procedure yields the rate

$$\tau^{-1} \sim \left[\left(\frac{\lambda T}{t\Delta_0}\right)^3\frac{\omega}{T}\right]^{1/5} T,\quad(6)$$

and in general there might be other regimes of higher ω leading to different temperature dependences. Observe that neither this rate nor (4) reverts to a cubic temperature dependence when substituting T for ω.

An important exclusion from the above generic situation is provided by the case of el-el interactions mediated by antiferromagnetic spin fluctuations, whose ordering vector is commensurate with the distance between the opposite pairs of nodes. In this case one arrives at a non-singular second order result $\tau^{-1} \sim \omega^{1/2} T^{5/2}$.

Although it is widely believed that this particular channel is relevant for the cuprates, it remains to be seen whether the lowest order estimate captures all the relevant physics. A formally related example of the system of interacting Dirac fermions describing quantum Hall plateau transitions [6] indicates that it may indeed be the case.

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